Novelty Journals

ISSN 2394-9651

International Journal of Novel Research in Physics Chemistry & Mathematics
Vol. 9, Issue 3, pp: (39-60), Month: September - December 2022, Available at: www.noveltyjournals.com

Fractional investigation of time derivatives airflow process in a rectangular building

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DOI: https://doi.org/10.5281/zenodo.7296349

Published Date: 06-November-2022

Abstract: **The study investigates the fractional time derivatives of air flow process across vertical openings in rectangular building by Caputo sense. Governing equations of air flow are solved analytically by Laplace transform technique and method of undetermined coefficient to obtain the solutions in Laplace domain. The solutions are inverted from the Laplace domain back to the time domain by the Riemann-sum approximation approach. The influence of the different flow parameters such as fractional order(α), Prandtl number (Pr), effective thermal coefficientθ_0 and discharge coefficient (c_d) are plotted in graphs. In the course of investigation, it is found that the increase or decrease in fractional order (α) between the interval of $0 < \alpha < 1$ the temperature profile and velocity profile will increase or decrease while the volumetric air flow and mass transfer will only decrease as fractional order increase.**

Keywords: **airflow process, building, fractional derivatives, investigations, rectangular, time.**

I. INTRODUCTION

Physical ventilation occurs not modern. It exists only in the preceding hundred years since mechanical ventilation brings into the globe subsisted initiated. Ahead of that period, all structure envelopes inhabited by humans subsisted as naturally circulated. The first biological ventilation aim can maybe be considered as the duration when these envelopes commence becoming purpose-building. Timely plans were mostly practical and developed from knowledge. They might virtually be interpreted as a long-term investigation of the full hierarchy. In many countries, traditional passive cooling strategies have been formulated alongside natural ventilation. Vastly of the experience gained over the centuries can be comprehended in unique natural-ventilation buildings. A reasonable illustration of this is the structure of wind-assisted machines. Nonetheless, contemporary are stressing electricity and energy consumption; criteria for health and satisfaction have to live met, while simultaneously alleviating codes for low energy consumption and sustainability. Normal ventilation is the method of replenishing and discarding air from an indoor opening to the ambient by realistic averages. These can either result in wind-driven or stack-drive outcomes. The goals of natural ventilation in the building exist to recoup energy consumption, donate thermal satisfaction, eliminated air breakdown increases indoor climate quality, with a reasonable structure ethical to the building area, and the aim of realistic ventilation can restore all or a fraction of the automatic ventilation network. Both struggle on the principle of the atmosphere progressing from a high-pressure to a low-pressure region. Work possesses existed achieved by various investigators, in the areas of Fluid dynamics on natural ventilation[1] Performed an experimental investigation on scale effect in room airflow and then later [2] Studied Air movement on natural ventilated building. [3] Investigated the effect of Stack driven airflow across two openings. [4] Investigated the effect of indirect flow velocity in rectangular building across three 6] Studied buoyancy-driven natural ventilation of buildings-

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impact of computational domain.[7] Considered the mixed ventilation when the interior of the building has uniform temperature. [8] Investigated displacement ventilation verticalopenings. [5] Performed an experiment in a room airflow distribution system using computational fluid dynamic(CFD). [when the interior of the building was stratified. [9] Examined the effect buoyancy- driven forces of airflow on the floor of an enclosure in the presence of wind. [10] Developed a modelwith the effect of wind forces.[11] Examined the heat- transfer and airflow in interactive building façade. [12]Simulations were carried out to reproduce the decay of $CO2$ concentration in a large semi-enclosed stadium. [13] Investigated natural convection flow for heat and mass- transfer in a single- sided ventilated building. [14] Considered ventilation potential model of airflow in china building. [15] Investigatedtransient airflow across two verticalupper-vents in the absent of opposing flow in rectangular building. Experimentwere performed in heated, sealed room of a test house by [16]. [17] Investigated a vertical temperature distribution by combining natural and mechanical ventilation in an atrium building. [18] investigatedthe effect of Stack- driven forces in single- sided ventilated building with large openings. [19] Investigated the effect of wind- driven flow in domain interacted with wind. [20] Investigated the effect of wind- driven flow in building with small openings. [21] Investigated the effect of two buoyancy forces on natural ventilation in an enclosure. [22] provided a book on introduction to fractional calculus.[23] studied the application of time fractional derivative with nonsingular kernel to the generalized convective of cassonfluid in micro channel with constant walls temperature.[24] studied fractional partial differential equation and their numerical solution.[25]considered an application of Caputo-Fabrizio derivatives to MHD free convection flow of generalized Walters'-B fluid model.[26] gives a comprehensive report on convective flow of fractional (ABC) and (CF) MHD viscous fluid subject to generalized boundary conditions.[27]studied a numerical investigation of fractional-order unsteady natural convective radiating flow of nanofluid in a vertical channel.[28]considered flows between two parallel plates of couple stress fluids with time-fractional caputo and caputo-fabrizio derivatives.[29]developedamodern approach of Caputo– Fabrizio time-fractional derivative to MHD free convection flow of generalized second-grade fluid in a porous medium.[30]considered a two- dimensional numerical simulation of natural ventilation in a multi- span greenhouse.

II. BODY OF ARTICLE

Building Description

The building to be considered is un-stratified Rectangular cross- ventilated with two (2) openings. In which the building has One (1) upper and one (1) lower rectangular opening. The upper openings have an area of $0.7m \times 1.0m$ respectively while the lower opening is $0.7m \times 2.0m$. Dimension of the domain is $5.3m \times 3.6m \times 2.8m$ with air as the connecting fluid. The domain envelops were separated from one another by a vertical rectangular opening of height$y$ and widthx_w, which is illustrated in Fig. 2.1. The density of air in the domain is maintained at ρ_0 with temperature at$T$ and pressure P.Schematic diagrams of airflow in one of the upper openings is also illustrated in Fig. 2.2 below it which, the flow is transient that depends on the height of the opening on the vertical walls.

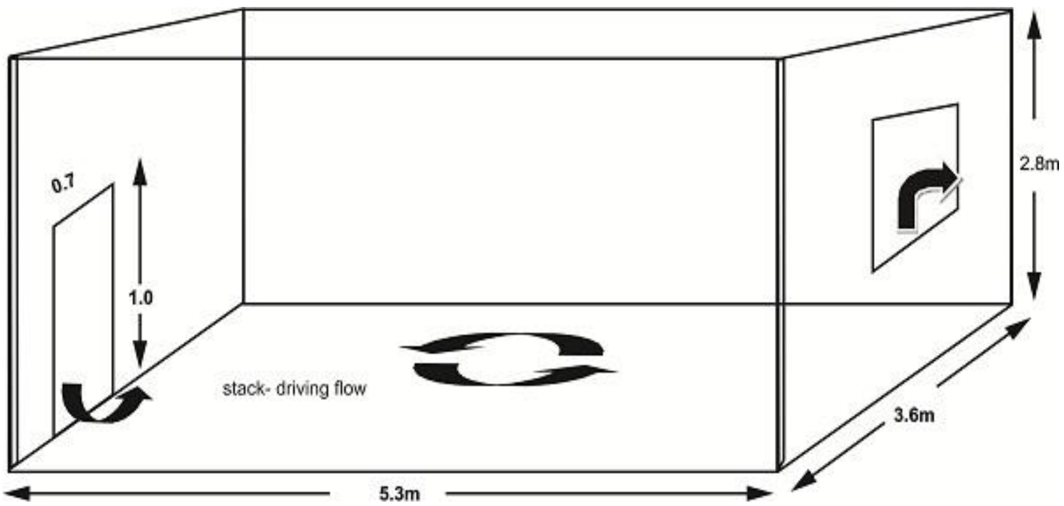

Fig.2.1: Diagram of un-stratified cross ventilated rectangular domain with two (2) openings.

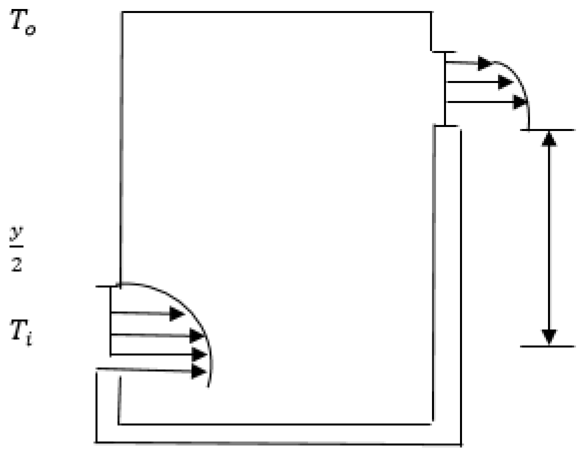

Fig. 2.2: Causing airflow through the vertical upper vent in the domain.

Governing Equations

Convective motion induced by buoyancy effect as illustrated in Fig. 2.1in which, described by the conservation equations for continuity, momentum and energy Equations (i.e. Navier- Stokes Equation) as,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad 2.1$$

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho_0}\frac{\partial P}{\partial x}-g+\frac{\mu}{\rho_0}\frac{\partial^2 u}{\partial y^2} \quad 2.2$$

$$\frac{\partial T}{\partial t}=\frac{k}{c_p\rho_0}\frac{\partial^2 T}{\partial y^2}+\frac{q}{c_p\rho_0}\emptyset_v \quad 2.3$$

Using the above preliminary assumptions, Equation (2.1), (2.2), (2.3) reduces to,

$$\frac{\partial u}{\partial x_w}+\frac{\partial v_0}{\partial y}=0 \quad 2.4$$

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho_0}\frac{\partial P}{\partial x_w}-g+\frac{\mu}{\rho_0}\frac{\partial^2 u}{\partial y^2} \quad 2.5$$

$$\frac{\partial T}{\partial t}=\frac{k}{c_p\rho_0}\frac{\partial^2 T}{\partial y^2} \quad 2.6$$

The pressure gradient given by,

$$-\frac{\partial p}{\partial x_w}=\rho_i g \quad 2.7$$

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial x_w}=\frac{g\rho_i}{\rho_0}$$

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial x_w}-g=\frac{g\rho_i}{\rho_0}- \qquad 2.8$$

Equation (2.5) becomes,

$$\frac{-1}{\rho_0}\frac{\partial p}{\partial x_w}-g=\frac{g\rho_i-g\rho_0}{\rho_0}=\frac{g(\rho_i-\rho_0)}{\rho_0}$$

$$-\frac{1}{\rho_0}\frac{\partial \rho}{\partial x_w}-g=\frac{g\Delta\rho}{\rho_0} \quad 2.9$$

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Where, $\Delta\rho = \rho_i - \rho_0$

Introducing the effect of reduced gravity which is defined as the fractional change in density generated by temperature difference,

$g' = \frac{g\Delta\rho}{\rho_0} = \frac{g\Delta T}{T_0}$2.10

In which, $\Delta T = T_i - T_0$ and $\beta = \frac{1}{T_0}$2.11

Equation (2.9) reduces to,

$-\frac{1}{\rho_a}\frac{\partial\rho}{\partial x_w} - g = g\beta\Delta T$2.12

Inserting Equation (2.12) into Equation (2.5) yields to,

$\frac{\partial u}{\partial t} = g\beta\Delta T + \frac{\mu}{\rho_0}\frac{\partial^2 u}{\partial y^2}$2.14

Equation (2.4) satisfied continuity equation identically and $\upsilon = {}^{\mu}/_{\rho_0}, \alpha = {}^{k}/_{c_p\rho_0}$ Equations (2.14) and (2.6) becomes,

$\frac{\partial u}{\partial t} = g\beta\Delta T + \upsilon\frac{\partial^2 U}{\partial y^2}$2.15

$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial y^2}$2.16

with the following dimensional initial and boundary conditions

$0 \le y \le 2, t > 0, \rho_0 = const \ne 0, x_w = constant, u(0,t) = 0, u(2,t) = 0, T(0,t) = -\theta_0, T(2,t) = 1 - \theta_0$17

1.4 Scaling

Introducing the non-dimensional quantities as

$y = y^*L, u = \frac{u^* g\beta\Delta T^* L^2}{\upsilon}, t = \frac{t^* L^2}{\alpha},\ T = T^*\Delta T^* + T_0,\ \Delta T = T - T_0$2.18

By putting Equation (2.18) into (2.15) yields to,

$\frac{\partial\left(\frac{u^* g\beta\Delta T^* L^2}{\alpha}\right)}{\partial\left(\frac{t^* L^2}{\upsilon}\right)} = g\beta T^*\Delta T^* + \upsilon\frac{\partial^2\left(\frac{u^* g\beta\Delta T^* L^2}{\alpha}\right)}{\partial(y^*L)^2}$2.19

$$\frac{\upsilon g\beta\Delta T^* L^2}{\alpha L^2}\frac{\partial u^*}{\partial t^*} = g\beta T^*\Delta T^* + \frac{\upsilon g\beta\Delta T^* L^2}{\alpha L^2}\frac{\partial^2 u^*}{y^{*2}}$$

$$\frac{\upsilon}{\alpha}\frac{\partial u^*}{\partial t^*} = T^* + \frac{\upsilon}{\alpha}\frac{\partial^2 u^*}{y^{*2}}$$

$$Pr\frac{\partial u^*}{\partial t^*} = Pr\frac{\partial^2 u^*}{\partial y^{*2}} + T^*$$

$\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{1}{Pr}T^*$2.20

By putting Equation (2.18) into (2.16) yields to,

$\frac{\partial(T^*\Delta T^* + T_0)}{\partial\left(\frac{t^* L^2}{\upsilon}\right)} = \alpha\frac{\partial^2(T^*\Delta T^* + T_0)}{\partial(y^*L)^2}$2.21

$$\frac{\upsilon\Delta T^*}{L^2}\frac{\partial T^*}{\partial t^*} = \frac{\Delta T^*\alpha}{L^2}\frac{\partial^2 T^*}{\partial y^{*2}}$$

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$$\frac{\upsilon}{\alpha}\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial y^{*2}}$$

$Pr\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial y^{*2}}$2.22

Where, $\frac{\upsilon}{\alpha} = Pr$

with the following dimensionless initial and boundary conditions

$0 \le y^* \le 1, t^* > 0, u^*(0,t^*) = 0, u^*(1,t^*) = 0, T^*(0,t^*) = -\theta_0, T^*(1,t^*) = 1 - \theta_0$2.23 The equations in (2.20) and (2.22) are transformed into fractional differential equations as seen below.

Fractional -Time derivative: Kirtiwantet. al. (2017), reported that for m to be smallest integer that exceedα, the Caputo fractional time derivative of order $\alpha > 0$ is defined as

$$D_t^\alpha u \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\tau)^{m-\alpha-1}\frac{\partial^m}{\partial t^m}u(x,\tau)\,d\tau\,, & form-1<\alpha\le m, \\ \frac{\partial^m}{\partial t^m}u(x,\tau) for \alpha = m\,, & m\in N \end{cases} \quad 2.23$$

Applying the Caputo fractional time derivative operator on equation (20) and (22) we obtained the following governing equations with their non-dimensional parameters as;

The dimensionless momentum equation:

$$D_t^\alpha u^*(y^*,t^*) = \frac{\partial^2 u^*(y^*,t^*)}{\partial y^{*2}} + \frac{1}{Pr}T^*(y^*,t^*) \quad 2.24$$

Is valid for $0 < \alpha < 1$ and with its initial and boundary condition as;

For $0 \le y^* \le 1$ and

$$t^* > 0: \begin{cases} u^*(y^*,t^*) = 0\,, & at y^* = 0 \\ u^*(y^*,t^*) = 0, & at y^* = 1 \end{cases} \quad 2.25$$

The dimensionless energy equation:

$$Pr D_t^\alpha T^*(y^*,t^*) = \frac{\partial^2 T^*(y^*,t^*)}{\partial y^{*2}} \quad 2.26$$

Is valid for $0 < \alpha < 1$ and with its initial and boundary condition as;

For $0 \le y^* \le 1$ and

$$t^* > 0: \begin{cases} T^*(y^*,t^*) = -\theta_0\,, & at y^* = 0 \\ T^*(y^*,t^*) = 1-\theta_0, & at y^* = 1 \end{cases} \quad 2.27$$

Where, Prandtl number(Pr) is dimensionless parameter, y^* is the dimensionless height of the opening, u^* is the dimensionless velocity of the flow, T^* is the dimensionless temperature of the flow, t^* is the dimensionless time and θ_0 is the effective thermal coefficient.

SOLUTION METHODS

This section (2) intends to describe the solution method of the dimensionless model equations (energy and momentum equations) in which the behaviour of parameters involved in the equations will predicts the temperature profile, velocity profile together with volumetric airflow and mass transfer.

2.1 Solution of the dimensionless energy equation

Taking the Laplace transform of Equation (26) subject to the initial and boundary condition in Equations (2.27) yields to;

$$\mathcal{L}\left\{\frac{1}{Pr}\frac{\partial^2 T^*(y^*,t^*)}{\partial y^{*2}}\right\} - \mathcal{L}\left\{\frac{\partial^\alpha T^*(y^*,t^*)}{\partial t^\alpha}\right\} = \mathcal{L}\{0\} \quad 2.28$$

$$\mathcal{L}\left\{\frac{1}{Pr}\frac{\partial^2 T^*(y^*,t^*)}{\partial y^{*2}}\right\} - \mathcal{L}\left\{\frac{\partial^\alpha T^*(y^*,t^*)}{\partial t^\alpha}\right\} = 0$$

$$\mathcal{L}\left\{\frac{\partial^\alpha T^*(y^*,t^*)}{\partial t^\alpha}\right\} = \mathcal{L}\left\{\frac{1}{Pr}\frac{\partial^2 T^*(y^*,t^*)}{\partial y^{*2}}\right\}$$

$\int_0^\infty e^{-sx}\frac{\partial^\alpha T^*}{\partial t^\alpha}dx = \frac{1}{Pr}\int_0^\infty e^{-sx}\frac{\partial^2 T^*}{\partial y^{*2}}dx$2.29

While its initial and boundary condition becomes;

$\frac{1}{s^2} > 0: \begin{cases} T^*(0,s^*) = -\frac{\theta_0}{s}, \; at y^* = 0 \\ T^*(1,s^*) = \frac{1-\theta_0}{s}, \; at y^* = 1 \end{cases}$ 2.30

Where, s is the Laplace parameter.

From the definition of Gamma functions, we have,

$\frac{\partial^\alpha T^*}{\partial t^{*\alpha}} = \frac{1}{\Gamma(m-\alpha)}\int_0^\infty (x-t^*)^{m-\alpha-1} T^{*m}(y^*,t^*)\, dt^*$2.31

Therefore,

$\int_0^\infty e^{-sx^*}\frac{\partial^\alpha T^*}{\partial t^{*\alpha}}dx^* = \int_0^\infty e^{-sx^*}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty (x^*-t^*)^{m-\alpha-1} T^{*m}(y^*,t^*)dt^*\right]dx^*$ 3.32

$= \int_0^\infty T^{*m}(y^*,t^*)dt^*\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty (x^*-t^*)^{m-\alpha-1}e^{-sx^*}\right]dx^*$2.33

From the Gamma function definition,

$$\Gamma(m) = \int_0^\infty x^{*m-1}e^{-x^*}dx^*$$

and let, $y^* = x^* - t,\; x^* = y^* + t^*, dx^* = dy^*$2.34

$\int_0^\infty T^{*m}(y^*,t^*)dt^*\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty (x^*-t^*)^{m-\alpha-1}e^{-sx^*}\right]dx^*$2.35

Inserting Equation (2.34) into (2.35) we get,

$$\int_0^\infty T^{*m}(y^*,t^*)dt^*\left[\frac{1}{(\Gamma m-\alpha)}\int_0^\infty y^{*m-\alpha-1}e^{-s(y^*+t^*)}\,dy^*\right]$$

$$= \int_0^\infty T^{*m}(y^*,t^*)dt^*\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{*m-\alpha-1}e^{-sy^*}e^{-st^*}\,dy^*\right]$$

$= \int_0^\infty T^{*m}(y^*,t^*)e^{-st^*}dt^*\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{m-\alpha-1}e^{-sy^*}dy^*\right]$2.36

$\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{m-\alpha-1}e^{-sy^*}dy^*$2.37

Applying integration by part to Equation (2.37),

$$f = y^{*m-\alpha-1}, dg = e^{-sy^*}dy^*, df = (m-\alpha-1)y^{*m-\alpha-2}dy^*, g = \frac{-1}{s}e^{-sy^*}$$

$$\int fdg = fg - \int gdf$$

$\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{m-\alpha-1}e^{-sy^*}dy^* = \frac{1}{\Gamma(m-\alpha)}\left[\left[\frac{-1}{s}e^{-sy^*}y^{m-\alpha-1}\right]_0^\infty + \int_0^\infty \frac{1}{s}(m-\alpha-1)y^{*m-\alpha-2}e^{-sy^*}dy^*\right]$2.38

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$$= \frac{1}{\Gamma(m-\alpha)}\left[0 + \frac{m-\alpha-1}{s}\int_0^\infty y^{*m-\alpha-2}\, e^{-sy^*}\, dy^*\right]$$

$= \frac{m-\alpha-1}{s\Gamma(m-\alpha)}\int_0^\infty y^{*m-\alpha-2}\, e^{-sy^*}\, dy^* 2.39$

$\int_0^\infty y^{*m-\alpha-2}\, e^{-sy^*}\, dy^* = \Gamma(m-\alpha-1) 2.40$

Inserting Equation (2.40) into (2.39) we have,

$\frac{m-\alpha-1}{s\Gamma(m-\alpha)}\Gamma(m-\alpha-1) 2.41$

From the Gamma properties,

$$\Gamma(m) = (m-1)\Gamma(m-1)$$

$\Gamma(m-\alpha) = (m-\alpha-1)\Gamma(m-\alpha-1) 2.42$

Putting Equation (2.42) into (2.41) yields to,

$\frac{\Gamma(m-\alpha)}{s\Gamma(m-\alpha)} = s^{-1} 2.43$

Is valid for $m-1 \le \alpha < m,\ m-1-\alpha \le 0$ implies $m-\alpha \le 1$

Inserting Equation (2.43) into (2.36) yields to,

$\int_0^\infty T^{*m}(y^*, t^*) e^{-st^*} s^{-1}\, dt^* 2.44$

But, $m-\alpha \le 1$

$\int_0^\infty T^{*m}(y^*, t^*) e^{-st^*} s^{-(m-\alpha)}\, dt^* 2.45$

$$\int_0^\infty T^{*m}(y^*, t^*) e^{-st^*} s^{-(m-\alpha)}\, dt^* = s^{-(m-\alpha)}\int_0^\infty T^{*m}(y^*, t^*) e^{-st^*}\, dt^*$$

$s^{-(m-\alpha)}\int_0^\infty T^{*m}(y^*, t^*) e^{-st^*}\, dt^* = s^{-(m-\alpha)}\left[s^m\overline{T^*}(y^*, s) - \sum_{k=0}^{m-1} s^{m-k-1}\,\overline{T^*}^k(0, s)\right] 2.46$

Where, $\int_0^\infty T^{*m}(t^*) e^{-st^*}\, dt^* = s^m\overline{T^*}(y^*, s) - \sum_{k=0}^{m-1} s^{m-k-1}\,\overline{T^*}^k(0, s)$

Is valid for $0 < \alpha < 1$

$$S^{-(m-\alpha)}\left[s^m\overline{T^*}(y^*, s) - \sum_{k=0}^{1-1} s^{m-k-1}\overline{T^*}^k(0, s)\right]$$

$= s^{-(m-\alpha)}\left[s^m\overline{T^*}(y^*, s) - s^{m-1}\overline{T^*}^0(0, s)\right] 2.47$

$$= s^{-(m-\alpha)}[s^m\overline{T^*}(y^*, s) - s^{m-1}]$$

$= s^{-(1-\alpha)}\left(s\overline{T^*}(y^*, s)\right) = s^{-1}s^\alpha\left(s\overline{T^*}(y^*, s)\right) \ = s^\alpha\overline{T^*}(y^*, s) 2.48$

When, $m = 1$

Therefore, Equation (2.29) becomes,

$Prs^\alpha\overline{T^*}(y^*, s) = \mathcal{L}\left\{\frac{\partial^2 T^*(y^*, t^*)}{\partial y^{*2}}\right\} 2.49$

Let, $\mathcal{L}\left\{\frac{\partial^2 T^*(y^*, t^*)}{\partial y^{*2}}\right\} = \frac{\partial^2\overline{T^*}(y^*, s)}{\partial y^{*2}}$

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$Prs^{\alpha}\overline{T^*}(y^*,s) = \frac{\partial^2\overline{T^*}(y^*,s)}{\partial y^{*2}}$2.50

$\overline{T^*}''(y^*,s) - Prs^{\alpha}\overline{T^*}(y^*,s) \ = 0$2.51

The Auxiliary Equation associated with the differential Equation in (2.51) is,

$$\overline{T^*} = Ae^{my^*}, \overline{T^*}' = \ Ame^{my^*}, \overline{T^*}'' = \ Am^2e^{my^*}$$

$(m^2 - Prs^{\alpha})Ae^{my^*} \ = 0$2.52

$$e^{my^*} \neq 0, A = 0, m^2 - Prs^{\alpha} = 0$$

Therefore,

$$m^2 = Prs^{\alpha}$$

$m = \pm\sqrt{Prs^{\alpha}}$2.53

The general solution of Equation (2.51) is,

$\overline{T^*}(y^*,s) = C_1e^{y^*\sqrt{Prs^{\alpha}}} + C_2e^{-y^*\sqrt{Prs^{\alpha}}}$2.54

Introducing hyperbolic functions identities, we have

$cosh\sqrt{Prs^{\alpha}}y^* = \frac{e^{y^*\sqrt{Prs^{\alpha}}}+e^{-y^*\sqrt{Prs^{\alpha}}}}{2}$, $sinh\sqrt{Prs^{\alpha}}y^* = \frac{e^{y^*\sqrt{Prs^{\alpha}}}-e^{-y^*\sqrt{Prs^{\alpha}}}}{2}$2.55

$$cosh\sqrt{Prs^{\alpha}}y^* + sinh\sqrt{Prs^{\alpha}}y^* \ = \frac{e^{y^*\sqrt{Prs^{\alpha}}}+e^{-y^*\sqrt{Prs^{\alpha}}}}{2} + \frac{e^{y^*\sqrt{Prs^{\alpha}}}-e^{-y^*\sqrt{Prs^{\alpha}}}}{2}$$

$$= e^{y^*\sqrt{Prs^{\alpha}}} \quad cosh\sqrt{Prs^{\alpha}}y^* - sinh\sqrt{Prs^{\alpha}}y^* \ = \frac{e^{y^*\sqrt{Prs^{\alpha}}}+e^{-y^*\sqrt{Prs^{\alpha}}}}{2} - \frac{e^{y^*\sqrt{Prs^{\alpha}}}-e^{-y^*\sqrt{Prs^{\alpha}}}}{2}$$

$$= e^{-y^*\sqrt{Prs^{\alpha}}}$$

From Equation (2.54) we have,

$$\overline{T^*}(y^*,s) = C_1\left(cosh\sqrt{Prs^{\alpha}}y^* + sinh\sqrt{Prs^{\alpha}}y^*\right) + C_2\left(cosh\sqrt{Prs^{\alpha}}y^* - sinh\sqrt{Prs^{\alpha}}y^*\right)$$

$$= C_1\,cosh\sqrt{Prs^{\alpha}}y^* + C_1\,sinh\sqrt{Prs^{\alpha}}y^* + C_2\,cosh\sqrt{Prs^{\alpha}}y^* - C_2\,sinh\sqrt{Prs^{\alpha}}y^*$$

$$= C_1\,cosh\sqrt{Prs^{\alpha}}y^* + C_1\,sinh\sqrt{Prs^{\alpha}}y^* + C_2\,cosh\sqrt{Prs^{\alpha}}y^* - C_2\,sinh\sqrt{Prs^{\alpha}}y^*$$

$$= (C_1 + C_2)\,cosh\sqrt{Prs^{\alpha}}\,y^* \ + \ (C_1 - C_2)sinh\sqrt{Prs^{\alpha}}y^*$$

Let, $K_1 = C_1 + C_2$, $K_2 = C_1 - C_2$

The general solution of Equation (54) becomes,

$\overline{T^*}(y^*,s) = K_1\,cosh\sqrt{Prs^{\alpha}}\,y^* + K_2 sinh\sqrt{Prs^{\alpha}}y^*$2.56

Applying the boundary conditions of Equation (2.30) in (2.56) we get,

$$T^*(0,s^*) = -\frac{\theta_0}{s} = K_1$$

$K_1 = -\frac{\theta_0}{s}$2.57

$$T^*(1,s^*) = \frac{1-\theta_0}{s} = -\frac{\theta_0}{s}cosh\sqrt{Prs^{\alpha}} + K_2 sinh\sqrt{Prs^{\alpha}}$$

$$= -\frac{\theta_0}{s}cosh\sqrt{Prs^{\alpha}} + K_2 sinh\sqrt{Prs^{\alpha}} = \frac{1}{s} - \frac{\theta_0}{s}$$

$$= -\frac{\theta_0}{s} cosh\sqrt{Prs^{\alpha}} + \frac{\theta_0}{s} + K_2 sinh\sqrt{Prs^{\alpha}} = \frac{1}{s}$$

$$\frac{\theta_0}{s}\left(1 - cosh\sqrt{Prs^{\alpha}}\right) + K_2 sinh\sqrt{Prs^{\alpha}} = \frac{1}{s}$$

$$K_2 sinh\sqrt{Prs^{\alpha}} = \frac{1}{s} - \frac{\theta_0}{s}\left(1 - cosh\sqrt{Prs^{\alpha}}\right)$$

$K_2 = \frac{1-\theta_0\left(1-cosh\sqrt{Prs^{\alpha}}\right)}{ssinh\sqrt{Prs^{\alpha}}}$2.58

Inserting Equation (2.57) and (2.58) into (2.56) we obtains the temperature profile as,

$\overline{T^*}(y^*, s) = -\frac{\theta_0}{s} cosh\sqrt{Prs^{\alpha}}\, y^* + \frac{1-\theta_0\left(1-cosh\sqrt{Prs^{\alpha}}\right)}{ssinh\sqrt{Prs^{\alpha}}} sinh\sqrt{Prs^{\alpha}} y^*$2.59

2.2 Solution of the dimensionless momentum equation

Taking the Laplace transform of Equation (2.24) subject to the initial and boundary condition in Equations (2.25) yields to;

$\mathcal{L}\left\{\frac{\partial^2 u^*(y^*,t^*)}{\partial y^{*2}}\right\} - \mathcal{L}\left\{\frac{\partial^{\alpha} u^*(y^*,t^*)}{\partial t^{\alpha}}\right\} = -\frac{1}{Pr}\mathcal{L}\{T^*(y^*, t^*)\}$ 2.60

$$\mathcal{L}\left\{\frac{\partial^2 u^*(y^*,t^*)}{\partial y^{*2}}\right\} = \mathcal{L}\left\{\frac{\partial^{\alpha} u^*(y^*,t^*)}{\partial t^{\alpha}}\right\} - \frac{1}{Pr}\mathcal{L}\{T^*(y^*, t^*)\}$$

$\mathcal{L}\left\{\frac{\partial^{\alpha} u^*(y^*,t^*)}{\partial t^{\alpha}}\right\} = \mathcal{L}\left\{\frac{\partial^2 u^*(y^*,t^*)}{\partial y^{*2}}\right\} + \frac{1}{Pr}\overline{T^*}(y^*, s)$2.61

Where, $\mathcal{L}\{T^*(y^*, t^*)\} = \overline{T^*}(y^*, s)$

While its initial and boundary condition becomes;

$\frac{1}{s^2} > 0$: $\begin{cases} u^*(0, s^*) = 0\,, & aty^* = 0 \\ u^*(1, s^*) = 0, & aty^* = 1 \end{cases}$ 2.62

$\mathcal{L}\left\{\frac{\partial^{\alpha} u^*(y^*,t^*)}{\partial t^{\alpha}}\right\} = \int_0^{\infty} e^{-sx} \frac{\partial^{\alpha} u^*}{\partial t^{\alpha}} dx$2.63

From the definition of Gamma functions we have,

$\frac{\partial^{\alpha} u^*}{\partial t^{*\alpha}} = \frac{1}{\Gamma(m-\alpha)}\int_0^{\infty}(x - t^*)^{m-\alpha-1} u^{*m}(y^*, t^*)\, dt^*$2.64

Therefore,

$\int_0^{\infty} e^{-sx^*} \frac{\partial^{\alpha} u^*}{\partial t^{*\alpha}} dx^* = \int_0^{\infty} e^{-sx^*}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^{\infty}(x^* - t^*)^{m-\alpha-1} u^{*m}(y^*, t^*) dt^*\right] dx^*$2.65

$= \int_0^{\infty} u^{*m}(y^*, t^*) dt^* \left[\frac{1}{\Gamma(m-\alpha)}\int_0^{\infty}(x^* - t^*)^{m-\alpha-1} e^{-sx^*}\right] dx^*$2.66

From the Gamma function definition,

$$\Gamma(m) = \int_0^{\infty} x^{*m-1} e^{-x^*} dx^*$$

and let, $y^* = x^* - t,\ x^* = y^* + t^*, dx^* = dy^*$2.67

$\int_0^{\infty} u^{*m}(y^*, t^*) dt^* \left[\frac{1}{\Gamma(m-\alpha)}\int_0^{\infty}(x^* - t^*)^{m-\alpha-1} e^{-sx^*}\right] dx^*$2.68

Inserting Equation (2.67) into (2.68) we get,

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$$\int_0^\infty u^{*m}(y^*,t^*)dt^*\left[\frac{1}{(\Gamma m-\alpha)}\int_0^\infty y^{*m-\alpha-1}e^{-s(y^*+t^*)}\,dy^*\right]$$

$$=\int_0^\infty u^{*m}(y^*,t^*)dt^*\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{*m-\alpha-1}e^{-sy^*}e^{-st^*}\,dy^*\right]$$

$= \int_0^\infty u^{*m}(y^*,t^*)e^{-st^*}\,dt^*\left[\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{m-\alpha-1}\,e^{-sy^*}dy^*\right]$2.69

$\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{m-\alpha-1}\,e^{-sy^*}dy^*$2.70

Applying integration by part to Equation (2.70),

$$a=y^{*m-\alpha-1}, db=e^{-sy^*}dy^*, da=(m-\alpha-1)y^{*m-\alpha-2}\,dy^*, b=\frac{-1}{s}e^{-sy^*}$$

$$\int adb=ab-\int bda$$

$\frac{1}{\Gamma(m-\alpha)}\int_0^\infty y^{m-\alpha-1}\,e^{-sy^*}dy^*=\frac{1}{\Gamma(m-\alpha)}\left[\left[\frac{-1}{s}e^{-sy^*}y^{m-\alpha-1}\right]_0^\infty+\int_0^\infty\frac{1}{s}(m-\alpha-1)y^{*m-\alpha-2}\,e^{-sy^*}\,dy^*\right]$2.71

$$=\frac{1}{\Gamma(m-\alpha)}\left[0+\frac{m-\alpha-1}{s}\int_0^\infty y^{*m-\alpha-2}\,e^{-sy^*}\,dy^*\right]$$

$=\frac{m-\alpha-1}{s\Gamma(m-\alpha)}\int_0^\infty y^{*m-\alpha-2}\,e^{-sy^*}\,dy^*$2.72

$\int_0^\infty y^{*m-\alpha-2}\,e^{-sy^*}\,dy^*=\Gamma(m-\alpha-1)$2.73

Inserting Equation (2.73) into (2.72) we have,

$\frac{m-\alpha-1}{s\Gamma(m-\alpha)}\Gamma(m-\alpha-1)$2.74

From the Gamma properties,

$$\Gamma(m)=(m-1)\Gamma(m-1)$$

$\Gamma(m-\alpha)=(m-\alpha-1)\Gamma(m-\alpha-1)$2.75

Putting Equation (2.75) into (2.74) yields to,

$\frac{\Gamma(m-\alpha)}{s\Gamma(m-\alpha)}=s^{-1}$2.76

Is valid for $m-1\le\alpha<m,\ m-1-\alpha\le 0$ implies $m-\alpha\le 1$

Inserting Equation (2.76) into (2.69) yields to,

$\int_0^\infty u^{*m}(y^*,t^*)e^{-st^*}s^{-1}\,dt^*$2.77

But, $m-\alpha\le 1$

$\int_0^\infty u^{*m}(y^*,t^*)e^{-st^*}s^{-(m-\alpha)}\,dt^*$2.78

$$\int_0^\infty u^{*m}(y^*,t^*)e^{-st^*}s^{-(m-\alpha)}\,dt^*=s^{-(m-\alpha)}\int_0^\infty u^{*m}(y^*,t^*)e^{-st^*}\,dt^*$$

$s^{-(m-\alpha)}\int_0^\infty u^{*m}(y^*,t^*)e^{-st^*}\,dt^*=s^{-(m-\alpha)}\left[s^m\overline{u^*}(y^*,s)-\sum_{k=0}^{m-1}s^{m-k-1}\overline{u^*}^k(0,s)\right]$ 2.79

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Where, $\int_0^\infty u^{*m}(t^*)e^{-st^*}\,dt^* = s^m\overline{u^*}(y^*,s) - \sum_{k=0}^{m-1} s^{m-k-1}\,\overline{u^*}^k(0,s)$

Is valid for $0 < \alpha < 1$

$$S^{-(m-\alpha)}\left[s^m\overline{u^*}(y^*,s) - \sum_{k=0}^{1-1} s^{m-k-1}\overline{u^*}^k(0,s)\right]$$

$= s^{-(m-\alpha)}\left[s^m\overline{u^*}(y^*,s) - s^{m-1}\overline{u^*}^0(0,s)\right]$2.80

$$= s^{-(m-\alpha)}[s^m\overline{u^*}(y^*,s) - s^{m-1}]$$

$= s^{-(1-\alpha)}\left(s\overline{u^*}(y^*,s)\right) = s^{-1}s^\alpha\left(s\overline{u^*}(y^*,s)\right) = s^\alpha\overline{u^*}(y^*,s)$2.81

When, $m = 1$

Therefore, Equation (2.61) becomes,

$s^\alpha\overline{u^*}(y^*,s) = \mathcal{L}\left\{\frac{\partial^2 u^*(y^*,t^*)}{\partial y^{*2}}\right\} + \frac{1}{Pr}\overline{T^*}(y^*,s)$2.82

Let, $\mathcal{L}\left\{\frac{\partial^2 u^*(y^*,t^*)}{\partial y^{*2}}\right\} = \frac{\partial^2\overline{u^*}(y^*,s)}{\partial y^{*2}}$

$$s^\alpha\overline{u^*}(y^*,s) = \frac{\partial^2\overline{u^*}(y^*,s)}{\partial y^{*2}} + \frac{1}{Pr}\overline{T^*}(y^*,s)$$

$\frac{\partial^2\overline{u^*}(y^*,s)}{\partial y^{*2}} - s^\alpha\overline{u^*}(y^*,s) = -\frac{1}{Pr}\overline{T^*}(y^*,s)$2.83

$\overline{u^*}''(y^*,s) - s^\alpha\overline{u^*}(y^*,s) \;= -\frac{1}{Pr}\left(-\frac{\theta_0}{s}\cosh\sqrt{Prs^\alpha}\,y^* + \frac{1-\theta_0\left(1-\cosh\sqrt{Prs^\alpha}\right)}{s\sinh\sqrt{Prs^\alpha}}\sinh\sqrt{Prs^\alpha}\,y^*\right)$2.84

The corresponding homogeneous Equation associated with the D.E of (2.84) is,

$\overline{u^*}''(y^*,s) - s^\alpha\overline{u^*}(y^*,s) \;= 0$2.85

The Auxiliary Equation associated with the differential Equation in (2.85) is,

$$\overline{u^*} = Be^{my^*}, \overline{u^*}' = Bme^{my^*}, \overline{u^*}'' = Bm^2e^{my^*}$$

$(m^2 - s^\alpha)Be^{my^*} = 0$2.86

$$e^{my^*} \neq 0, B = 0, m^2 - s^\alpha = 0$$

Therefore,

$$m^2 = s^\alpha$$

$m = \pm\sqrt{s^\alpha}$2.87

The complementary solution of Equation (2.85) is,

$\overline{u^*}_c(y^*,s) = C_3e^{y^*\sqrt{s^\alpha}} + C_4e^{-y^*\sqrt{s^\alpha}}$2.88

Introducing hyperbolic functions identities, we have

$\cosh\sqrt{s^\alpha}y^* = \frac{e^{y^*\sqrt{s^\alpha}} + e^{-y^*\sqrt{s^\alpha}}}{2}$, $\sinh\sqrt{s^\alpha}y^* = \frac{e^{y^*\sqrt{s^\alpha}} - e^{-y^*\sqrt{s^\alpha}}}{2}$2.89

$$\cosh\sqrt{s^\alpha}y^* + \sinh\sqrt{s^\alpha}y^* = \frac{e^{y^*\sqrt{s^\alpha}} + e^{-y^*\sqrt{s^\alpha}}}{2} + \frac{e^{y^*\sqrt{s^\alpha}} - e^{-y^*\sqrt{s^\alpha}}}{2} = e^{y^*\sqrt{s^\alpha}}$$

$$\cosh\sqrt{s^\alpha}y^* - \sinh\sqrt{s^\alpha}y^* = \frac{e^{y^*\sqrt{s^\alpha}} + e^{-y^*\sqrt{s^\alpha}}}{2} - \frac{e^{y^*\sqrt{s^\alpha}} - e^{-y^*\sqrt{s^\alpha}}}{2} = e^{-y^*\sqrt{s^\alpha}}$$

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From Equation (2.88) we have,

$$\overline{u^*}_c(y^*,s) = C_3\left(\cosh\sqrt{s^\alpha}y^* + \sinh\sqrt{s^\alpha}y^*\right) + C_4\left(\cosh\sqrt{s^\alpha}y^* - \sinh\sqrt{s^\alpha}y^*\right)$$

$$= C_3\cosh\sqrt{s^\alpha}y^* + C_3\sinh\sqrt{s^\alpha}y^* + C_4\cosh\sqrt{s^\alpha}y^* - C_4\sinh\sqrt{s^\alpha}y^*$$

$$= (C_3 + C_4)\cosh\sqrt{s^\alpha}\,y^* + (C_3 - C_4)\sinh\sqrt{s^\alpha}y^*$$

Let, $K_3 = C_3 + C_4$, $K_4 = C_3 - C_4$

The complementary solution of Equation (2.88) is,

$\overline{u^*}_c(y^*,s) = K_3\cosh\sqrt{s^\alpha}\,y^* + K_4\sinh\sqrt{s^\alpha}y^*$2.90

From Equation (2.84) our L.H.S is given by,

$$R(y^*) = -\frac{1}{Pr}\left(-\frac{\theta_0}{s}\cosh\sqrt{Prs^\alpha}\,y^* + \frac{1-\theta_0\left(1-\cosh\sqrt{Prs^\alpha}\right)}{s\sinh\sqrt{Prs^\alpha}}\sinh\sqrt{Prs^\alpha}y^*\right)$$

Therefore, the particular solution is given as,

$\overline{u^*}_p(y^*,s) = \overline{u^*}_{p_1}(y^*,s) + \overline{u^*}_{p_2}(y^*,s)$2.91

$$\overline{u^*}_{p_1}(y^*,s) = A_0\cosh\sqrt{Prs^\alpha}\,y^* + B_0\sinh\sqrt{Prs^\alpha}\,y^*$$

$$\overline{u^*}_{p_2}(y^*,s) = C_0\sinh\sqrt{Prs^\alpha}\,y^* + D_0\cosh\sqrt{Prs^\alpha}\,y^*$$

$$\overline{u^*}_p(y^*,s) = A_0\cosh\sqrt{Prs^\alpha}\,y^* + B_0\sinh\sqrt{Prs^\alpha}\,y^* + C_0\sinh\sqrt{Prs^\alpha}\,y^* + D_0\cosh\sqrt{Prs^\alpha}\,y^*$$

$$\overline{u^*}_p(y^*,s) = (A_0 + D_0)\cosh\sqrt{Prs^\alpha}\,y^* + (B_0 + C_0)\sinh\sqrt{Prs^\alpha}\,y^*$$

$\overline{u^*}_p(y^*,s) = E_0\cosh\sqrt{Prs^\alpha}\,y^* + F_0\sinh\sqrt{Prs^\alpha}\,y^*$2.92

Where, $E_0 = A_0 + D_0$ and $F_0 = B_0 + C_0$

Differentiating Equation (2.92) we have,

$$\overline{u^*}_p{}' = E_0\sqrt{Prs^\alpha}\sinh\sqrt{Prs^\alpha}\,y^* + F_0\sqrt{Prs^\alpha}\cosh\sqrt{Prs^\alpha}\,y^*$$

$\overline{u^*}_p{}'' = E_0 Prs^\alpha\cosh\sqrt{Prs^\alpha}\,y^* + F_0 Prs^\alpha\sinh\sqrt{Prs^\alpha}\,y^*$2.94

Substituting Equation (2.92) and (2.94) into (2.84) we have,

$$E_0 prs^\alpha\cosh\sqrt{Prs^\alpha}\,y^* + F_0 Prs^\alpha\sinh\sqrt{Prs^\alpha}\,y^* - s^\alpha\left(E_0\cosh\sqrt{Prs^\alpha}\,y^* + F_0\sinh\sqrt{Prs^\alpha}\,y^*\right) = -\frac{1}{Pr}\left(-\frac{\theta_0}{s}\cosh\sqrt{Prs^\alpha}\,y^* + \frac{1-\theta_0\left(1-\cosh\sqrt{Prs^\alpha}\right)}{s\sinh\sqrt{Prs^\alpha}}\sinh\sqrt{Prs^\alpha}y^*\right) \qquad 2.95$$

$$E_0 prs^\alpha\cosh\sqrt{Prs^\alpha}\,y^* + F_0 Prs^\alpha\sinh\sqrt{Prs^\alpha}\,y^* + E_0 s^\alpha\cosh\sqrt{Prs^\alpha}\,y^* - F_0 s^\alpha\sinh\sqrt{Prs^\alpha}\,y^*$$
$$= -\frac{1}{Pr}\left(-\frac{\theta_0}{s}\cosh\sqrt{Prs^\alpha}\,y^* + \frac{1-\theta_0\left(1-\cosh\sqrt{Prs^\alpha}\right)}{s\sinh\sqrt{Prs^\alpha}}\sinh\sqrt{Prs^\alpha}y^*\right)$$

$$E_0\cosh\sqrt{Prs^\alpha}\,y^*(Prs^\alpha - s^\alpha) + F_0\sinh\sqrt{Prs^\alpha}\,y^*(Prs^\alpha - s^\alpha)$$
$$= -\frac{1}{Pr}\left(-\frac{\theta_0}{s}\cosh\sqrt{Prs^\alpha}\,y^* + \frac{1-\theta_0\left(1-\cosh\sqrt{Prs^\alpha}\right)}{s\sinh\sqrt{Prs^\alpha}}\sinh\sqrt{Prs^\alpha}y^*\right)$$

$E_0\cosh\sqrt{Prs^\alpha}\,y^*(Prs^\alpha - s^\alpha) + F_0\sinh\sqrt{Prs^\alpha}\,y^*(Prs^\alpha - s^\alpha) = -\frac{1}{Pr}\left(G\cosh\sqrt{Prs^\alpha}\,y^* + H\sinh\sqrt{Prs^\alpha}y^*\right)$2.96

Where, $G = -\frac{\theta_0}{s}$ and $H = \frac{1-\theta_0\left(1-\cosh\sqrt{prs^\alpha}\right)}{s\sinh\sqrt{Prs^\alpha}}$

To find the arbitrary constants $E_0$and F_0 we have,

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$$cosh\sqrt{prs^{\alpha}}\,y^*: E_0(Prs^{\alpha}-s^{\alpha}) = -\frac{G}{Pr}$$

Therefore, $E_0 = -\frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}$2.97

$$sinh\sqrt{prs^{\alpha}}\,y^*: F_0(Prs^{\alpha}-s^{\alpha}) = -\frac{H}{Pr}$$

Therefore, $F_0 = -\frac{H}{Pr(Prs^{\alpha}-s^{\alpha})}$2.98

Inserting Equations (2.97) and (2.98) into (2.92) one obtains,

$\overline{u^*}_p(y^*,s) = -\frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{Prs^{\alpha}}\,y^* - \frac{H}{Pr(Prs^{\alpha}-s^{\alpha})}sinh\sqrt{Prs^{\alpha}}\,y^*$2.99

The general solution of (2.84) is of the form of,

$\overline{u^*}(y^*,s) = \overline{u^*}_c(y^*,s) + \overline{u^*}_p(y^*,s)$2.100

Inserting Equations (2.90) and (2.99) into (2.100) one obtain the general solution as,

$\overline{u^*}(y^*,s) = K_3\,cosh\sqrt{s^{\alpha}}\,y^* + K_4 sinh\sqrt{s^{\alpha}}y^* - \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{Prs^{\alpha}}\,y^* - \frac{H}{Pr(Prs^{\alpha}-s^{\alpha})}sinh\sqrt{Prs^{\alpha}}\,y^*$2.101

By applying the boundary condition of (2.62) in (2.101) we have,

$$\overline{u^*}(0,s) = 0 = K_3 - \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}$$

$K_3 = \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}$2.102

$$\overline{u^*}(1,s) = 0 = \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{s^{\alpha}} + K_4 sinh\sqrt{s^{\alpha}} - \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{Prs^{\alpha}} - \frac{H}{Pr(Prs^{\alpha}-s^{\alpha})}sinh\sqrt{Prs^{\alpha}}$$

$K_4 = \frac{G}{sinh\sqrt{s^{\alpha}}Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{Prs^{\alpha}} + \frac{H}{sinh\sqrt{s^{\alpha}}Pr(Prs^{\alpha}-s^{\alpha})}sinh\sqrt{Prs^{\alpha}} - \frac{G}{sinh\sqrt{s^{\alpha}}Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{s^{\alpha}}$2.103

Inserting Equation (2.102) and (2.103) into (2.101) we obtains the velocity profile as,

$\overline{u^*}(y^*,s) = \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{s^{\alpha}}\,y^* + \Big[\frac{G}{sinh\sqrt{s^{\alpha}}Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{Prs^{\alpha}} + \frac{H}{sinh\sqrt{s^{\alpha}}Pr(Prs^{\alpha}-s^{\alpha})}sinh\sqrt{Prs^{\alpha}} - \frac{G}{sinh\sqrt{s^{\alpha}}Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{s^{\alpha}}\Big] sinh\sqrt{s^{\alpha}}y^* - \frac{G}{Pr(Prs^{\alpha}-s^{\alpha})}cosh\sqrt{Prs^{\alpha}}\,y^* - \frac{H}{Pr(Prs^{\alpha}-s^{\alpha})}sinh\sqrt{Prs^{\alpha}}\,y^*$3.104

$\overline{u^*}(y^*,s) = \frac{\theta_0\left(cosh\sqrt{s^{\alpha}}y^* - cosh\sqrt{Prs^{\alpha}}y^*\right)}{sPr(Prs^{\alpha}-s^{\alpha})} + \frac{\theta_0\left(cosh\sqrt{Prs^{\alpha}} - cosh\sqrt{s^{\alpha}}\right)sinh\sqrt{s^{\alpha}}y^*}{sPr\,sinh\sqrt{s^{\alpha}}(Prs^{\alpha}-s^{\alpha})} + \frac{\left(sinh\sqrt{Prs^{\alpha}}y^* - \theta_0\,sinh\sqrt{Prs^{\alpha}}y^* + \theta_0\,cosh\sqrt{Prs^{\alpha}}\,sinh\sqrt{Prs^{\alpha}}y^*\right)}{sPr\,sinh\sqrt{Prs^{\alpha}}(Prs^{\alpha}-s^{\alpha})} - \frac{\left(sinh\sqrt{s^{\alpha}}y^* - \theta_0\,sinh\sqrt{s^{\alpha}}y^* + \theta_0\,cosh\sqrt{Prs^{\alpha}}\,sinh\sqrt{s^{\alpha}}y^*\right)}{sPr\,sinh\sqrt{s^{\alpha}}(Prs^{\alpha}-s^{\alpha})}$2.105

Equation (2.105) reduces to the velocity profile as,

$\overline{u^*}(y^*,s) = \frac{1}{sPrsinh\sqrt{s^{\alpha}}\,sinh\sqrt{Prs^{\alpha}}(Prs^{\alpha}-s^{\alpha})}\Big[\theta_0 sinh\sqrt{s^{\alpha}}\,sinh\sqrt{Prs^{\alpha}}\left(cosh\sqrt{s^{\alpha}}\,y^* - cosh\sqrt{Prs^{\alpha}}\,y^*\right) + \theta_0\,sinh\sqrt{Prs^{\alpha}}\left(cosh\sqrt{Prs^{\alpha}} - cosh\sqrt{s^{\alpha}}\right)sinh\sqrt{s^{\alpha}}\,y^* + sinh\sqrt{s^{\alpha}}\left(1-\theta_0+\theta_0\,cosh\sqrt{Prs^{\alpha}}\right)sinh\sqrt{Prs^{\alpha}}\,y^* + sinh\sqrt{Prs^{\alpha}}\left(\theta_0 - 1 - \theta_0\,cosh\sqrt{Prs^{\alpha}}\right)sinh\sqrt{s^{\alpha}}\,y^*\Big]$2.106

Volumetric airflow

From the elementary Physics,

$$Q(y^*,s) \propto A^* \int_{n=0}^{n=\frac{y^*}{2}} \overline{u^*}(n,s)dn$$

$Q(y^*,s) = A^* c_d \int_{n=0}^{n=\frac{y^*}{2}} \overline{u^*}(n,s) dn$2.107

$Q(y^*,s) = \frac{A^* c_d}{sPrsinh\sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}(Prs^\alpha - s^\alpha)} \int_{n=0}^{n=\frac{y^*}{2}} [\theta_0 sinh\sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}\left(cosh\sqrt{s^\alpha}\, n - cosh\sqrt{Prs^\alpha}\, n\right) +$
$\theta_0\, sinh\sqrt{Prs^\alpha}\left(cosh\sqrt{Prs^\alpha} - cosh\sqrt{s^\alpha}\right) sinh\sqrt{s^\alpha}\, n + sinh\sqrt{s^\alpha}\left(1 - \theta_0 + \theta_0\, cosh\sqrt{Prs^\alpha}\right) sinh\sqrt{Prs^\alpha}\, n +$
$sinh\sqrt{Prs^\alpha}\left(\theta_0 - 1 - \theta_0\, cosh\sqrt{Prs^\alpha}\right) sinh\sqrt{s^\alpha}\, n] dn$2.108

After taking the integral of Equation (108) one obtains the volumetric air flow as,

$Q(y^*,s) = \frac{A^* c_d}{sPrsinh\sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}(Prs^\alpha - s^\alpha)} \left[\theta_0 sinh\sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}\left(\sqrt{Prs^\alpha}\, sinh\sqrt{s^\alpha}\frac{y^*}{2} - \sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}\frac{y^*}{2}\right) +\right.$
$\theta_0\sqrt{Prs^\alpha}\, sinh\sqrt{Prs^\alpha}\left(cosh\sqrt{Prs^\alpha} - cosh\sqrt{s^\alpha}\right)\left(cosh\sqrt{s^\alpha}\frac{y^*}{2} - \sqrt{s^\alpha}\right) + \sqrt{s^\alpha}\, sinh\sqrt{s^\alpha}\left(1 - \theta_0 +\right.$
$\left.\theta_0\, cosh\sqrt{Prs^\alpha}\right)\left(cosh\sqrt{Prs^\alpha}\frac{y^*}{2} - \sqrt{Prs^\alpha}\right) + \sqrt{Prs^\alpha}\, sinh\sqrt{Prs^\alpha}\left(\theta_0 - 1 - \theta_0\, cosh\sqrt{Prs^\alpha}\right)\left(cosh\sqrt{s^\alpha}\frac{y^*}{2} -\right.$
$\left.\left.\sqrt{s^\alpha}\right)\right]$2.109

Mass transfer

From the elementary Physics,

$$\rho_0 = \frac{m(y^*,s)}{Q(y^*,s)}$$

$m(y^*,s) = \rho_0 Q(y^*,s)$2.110

Inserting Equation (2.109) into (2.110) yields to mass transfer as,

$m(y^*,s) = \frac{A^* \rho_0 c_d}{sPrsinh\sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}(Prs^\alpha - s^\alpha)} \left[\theta_0 sinh\sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}\left(\sqrt{Prs^\alpha}\, sinh\sqrt{s^\alpha}\frac{y^*}{2} - \sqrt{s^\alpha}\, sinh\sqrt{Prs^\alpha}\frac{y^*}{2}\right) +\right.$
$\theta_0\sqrt{Prs^\alpha}\, sinh\sqrt{Prs^\alpha}\left(cosh\sqrt{Prs^\alpha} - cosh\sqrt{s^\alpha}\right)\left(cosh\sqrt{s^\alpha}\frac{y^*}{2} - \sqrt{s^\alpha}\right) + \sqrt{s^\alpha}\, sinh\sqrt{s^\alpha}\left(1 - \theta_0 +\right.$
$\left.\theta_0\, cosh\sqrt{Prs^\alpha}\right)\left(cosh\sqrt{Prs^\alpha}\frac{y^*}{2} - \sqrt{Prs^\alpha}\right) + \sqrt{Prs^\alpha}\, sinh\sqrt{Prs^\alpha}\left(\theta_0 - 1 - \theta_0\, cosh\sqrt{Prs^\alpha}\right)\left(cosh\sqrt{s^\alpha}\frac{y^*}{2} -\right.$
$\left.\left.\sqrt{s^\alpha}\right)\right]$2.111

Where,$\overline{T^*}(y^*,s)$ is the temperature profile, $\overline{u^*}(y^*,s)$ is the velocity profile, $Q(y^*,s)$ is the volumetric airflow, A^* is the total area of the openings, c_d is the discharge coefficient and n is the dummy variable, $m(y^*,s)$ is the mass transfer, θ_0 is the effective thermal coefficient, y^* is the height of the openings, s is the Laplace parameter and ρ_0 is the interior density of air.

Equations (2.59), (2.106), (2.109) and (2.111) which are in Laplace domain need to be inverted by Riemann- sum approximation in order to determine the temperature profile, velocity profile together with volumetric air flow and mass transfer in time domain. Due to the difficulty in obtaining the inverse of these equations, we use a numerical means. In this method, functions in the Laplace domain "s" can be inverted to time domain as follows;

$T^*(y^*,t^*) = \frac{e^{\varepsilon t^*}}{t^*}\left(\frac{1}{2}\overline{T^*}(\varepsilon,y^*) + Re\sum_{k=1}^{n}(-1)^k \overline{T^*}\left(\varepsilon + \frac{ik\pi}{t^*}, y^*\right)\right)$2.112

$u^*(y^*,t^*) = \frac{e^{\varepsilon t^*}}{t^*}\left(\frac{1}{2}\overline{u^*}(\varepsilon,y^*) + Re\sum_{k=1}^{n}(-1)^k \overline{u^*}\left(\varepsilon + \frac{ik\pi}{t^*}, y^*\right)\right)$ 2.113

$Q(y^*,t^*) = \frac{e^{\varepsilon t^*}}{t^*}\left(\frac{1}{2}\bar{Q}(\varepsilon,y^*) + Re\sum_{k=1}^{n}(-1)^k \bar{Q}\left(\varepsilon + \frac{ik\pi}{t^*}, y^*\right)\right)$ 114

$m(y^*,t^*) = \frac{e^{\varepsilon t^*}}{t^*}\left(\frac{1}{2}\bar{m}(\varepsilon,y^*) + Re\sum_{k=1}^{n}(-1)^k \bar{m}\left(\varepsilon + \frac{ik\pi}{t^*}, y^*\right)\right)$ 115

Abiodun O. Ajibade (2013), clearly explained the usefulness of Equation (112) to obtained desirable result, where Re is the real part, $i = \sqrt{-1}$ is the imaginary part, n been the number of terms used in the Riemann-sum approximation and the real part of the Bromwich contour is ε which is used to invert functions in Laplace domain to time domain. The Riemann-sum

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approximation for the Laplace domain consists of a single summation for the numerical process of which it accuracy relies on the value of ε and the truncated error dictated by$n$. The value of ε should be selected so that the Bromwich contour encloses all the branch points. The numerical experiment that provides a faster convergence which gives the most satisfying result is$\varepsilon t^* \cong 4.7$. Therefore, the right value of ε for faster convergence depends on the instant of time (t^*) at which the lagging surrounding phenomenon is studied. The conditions reveal by $\varepsilon t^* \cong 4.7$ does not rely on the value of t^* but the number n of terms in the Riemann-sum is determined. Usually, using the given values ofy^*, $t^*$and ε the prescribed threshold for the dense accumulated partial sum is satisfied.

NUMERICAL SIMULATIONS AND DISCUSSION OF THE RESULTS

Numerical Simulation of the results

This present section discusses the results obtained from chapter three in form of line graphs that gives a proper explanation on the analytical results. The application of fractional time derivatives to air flow across vertical vents in rectangular domain have been studied with the aid of line graphs. In which the effect of parameters and other operating conditions involved in the study will be carry out for three (3) different values of parameters such as; $\alpha: 0.1, 0.3, 0.5$, $\theta_0: 0.01, 0.03, 0.05$, $Pr: 0.69, 0.71, 0.73$ and $c_a: 0.58, 0.60, 0.62$ are shown in Fig.2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10, 2.11, 2.12, 2.13, 2.14 and 2.15. This is done in order to see the effect of changes of parameters to the overall flow distributions,while keeping other operating conditions and parameters fixed, and ascertain the best one for optimal natural ventilation.

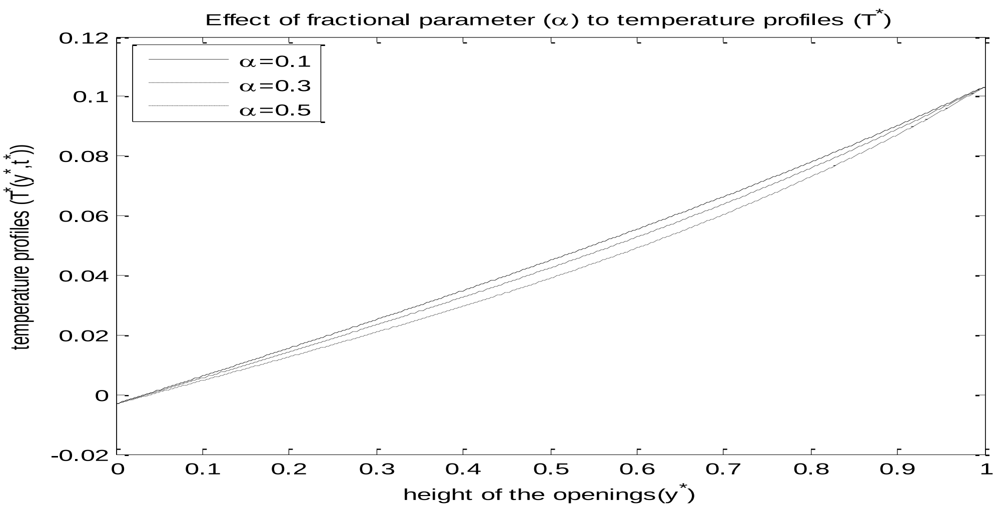

Fig. 2.3: Temperature profileT^*versus y^* and t^* for $\alpha = 0.1, 0.3, 0.5$

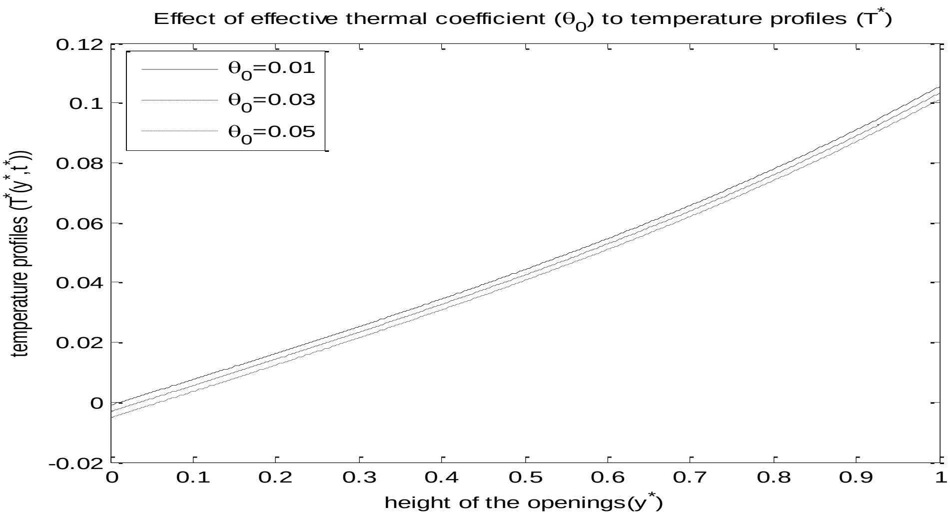

Fig.2.4: Temperature profileT^*versus y^* and t^* for$\theta_0 = 0.01, 0.03, 0.05$

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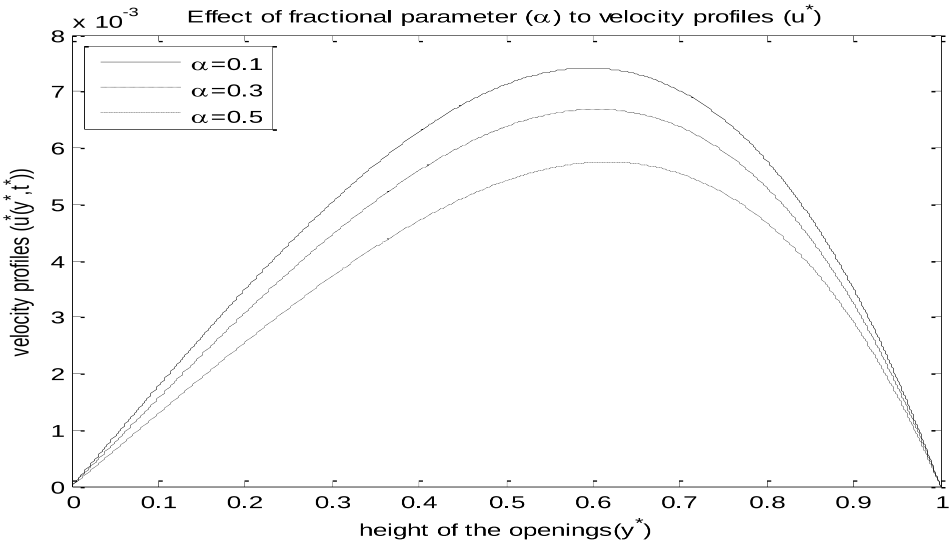

Fig. 2.5: velocity profileu^*versus y^* and t^* for $\alpha = 0.1, 0.3, 0.5$

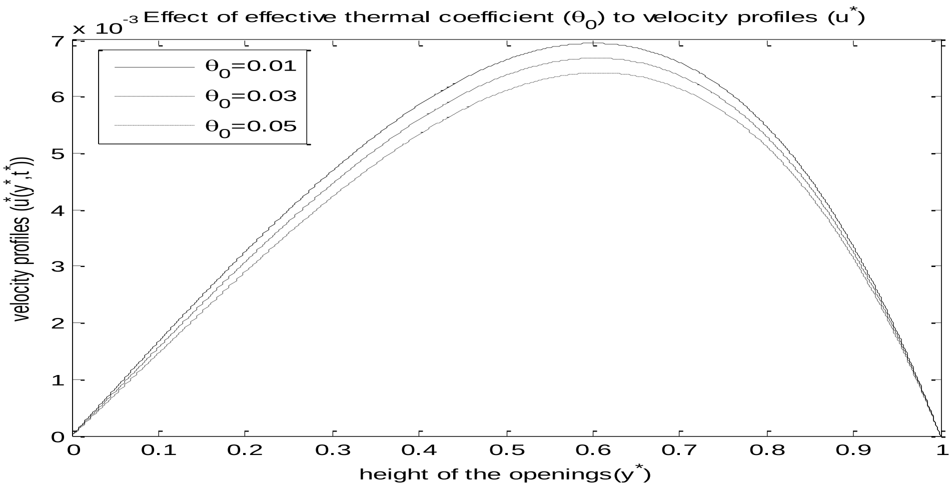

Fig. 2.6: velocity profileu^*versus y^* and t^* for$\theta_0 = 0.01, 0.03, 0.05$

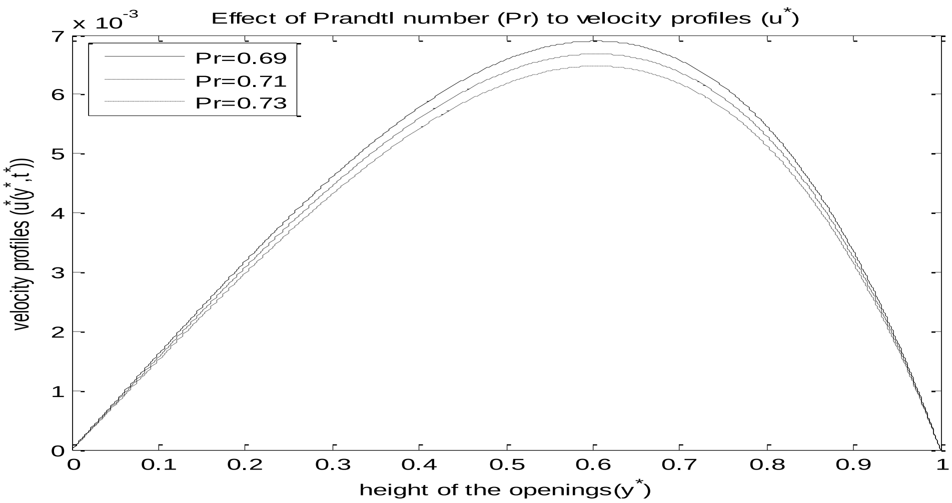

Fig.2.7: velocity profileu^*versus y^* and t^* for$Pr = 0.69, 0.71, 0.73$

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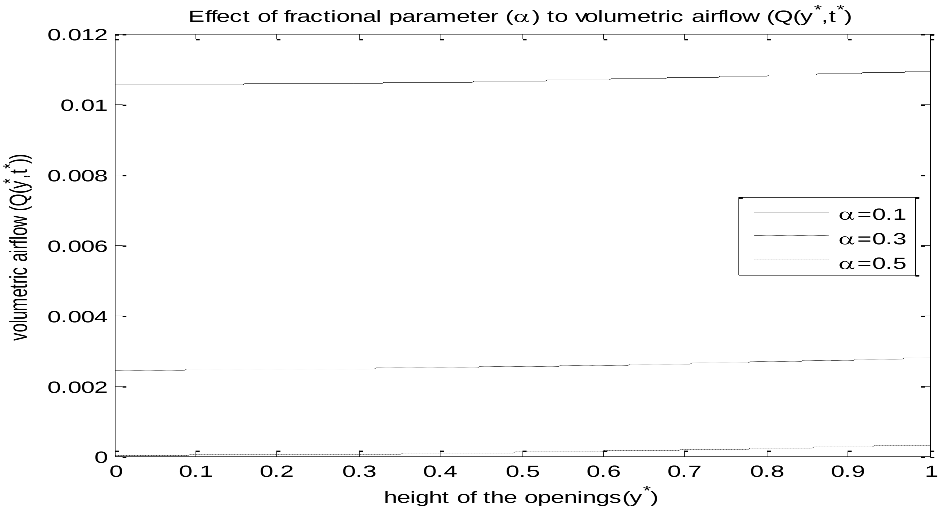

Fig. 2.8: volumetric airflow Qversus y^* and t^* for $\alpha = 0.1, 0.3, 0.5$

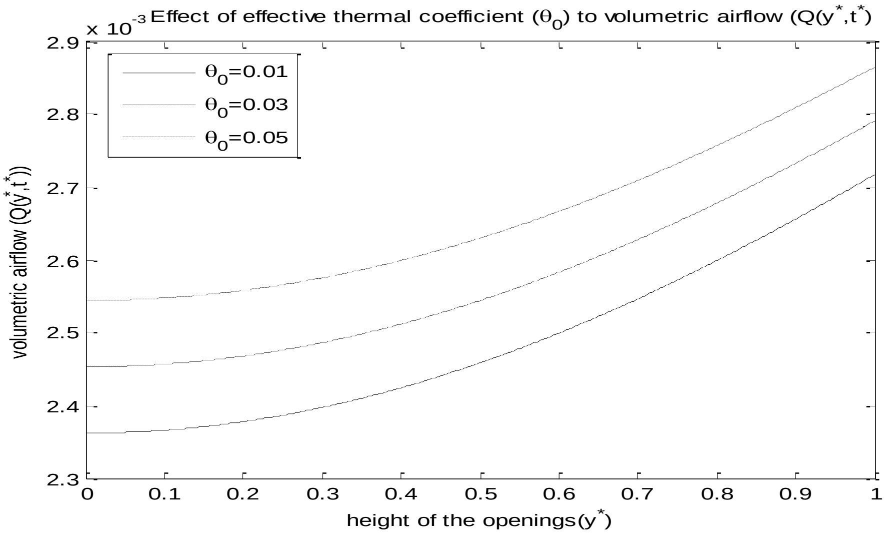

Fig.2.9: volumetric airflow Qversus y^* and t^* for$\theta_0 = 0.01, 0.03, 0.05$

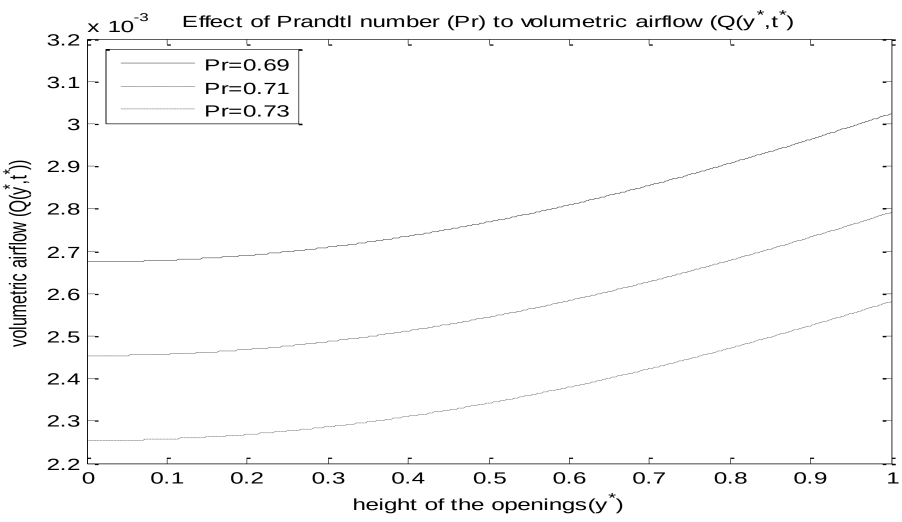

Fig. 2.10: volumetric airflow Qversus y^* and t^* for$Pr = 0.69, 0.71, 0.73$

ISSN 2394-9651
International Journal of Novel Research in Physics Chemistry & Mathematics
Vol. 9, Issue 3, pp: (39-60), Month: September - December 2022, Available at: www.noveltyjournals.com

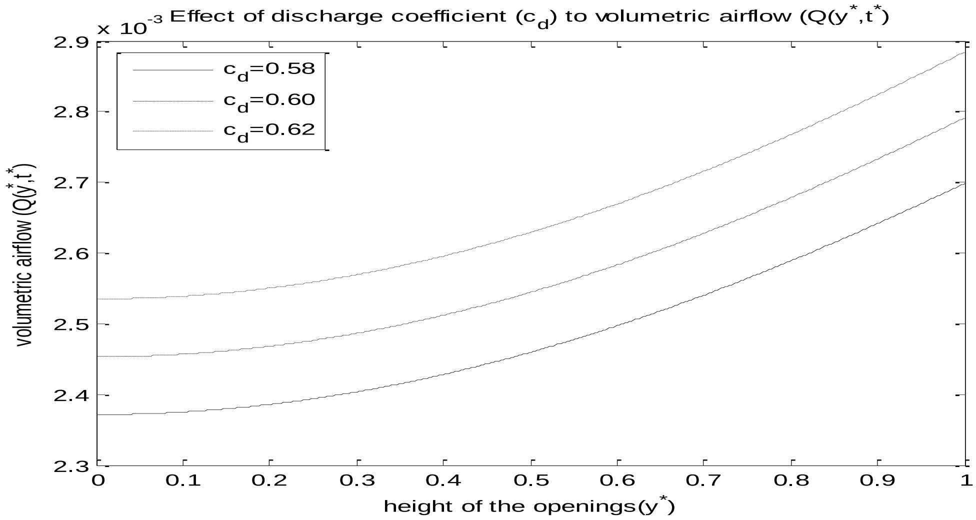

Fig. 2.11: volumetric airflow Qversus y^* and t^* for$c_d = 0.58, 0.60, 0.62$

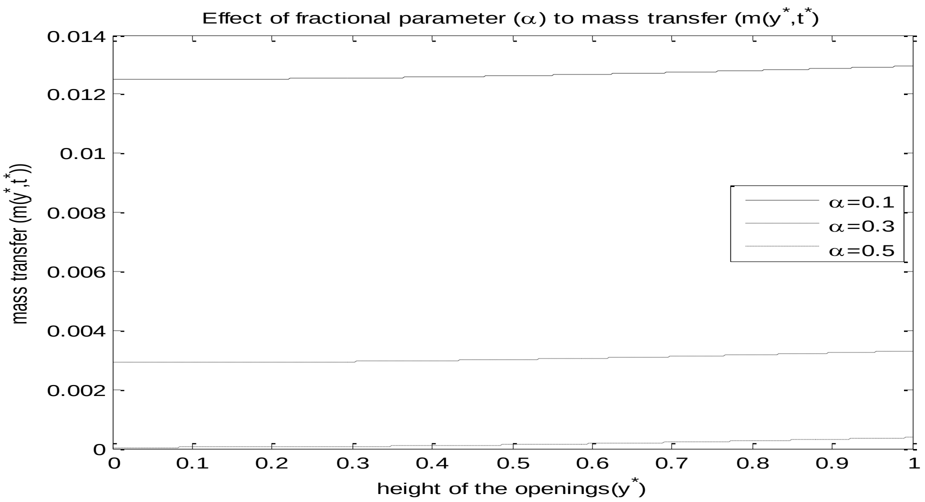

Fig. 2.12: mass transfer $m$versus y^* and t^* for $\alpha = 0.1, 0.3, 0.5$

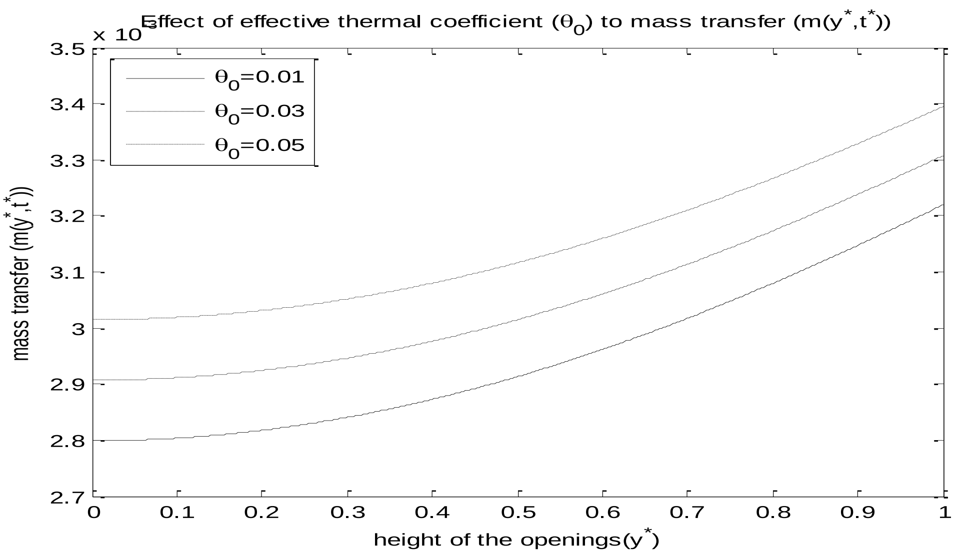

Fig. 2.13: mass transfer $m$versus y^* and t^* for$\theta_0 = 0.01, 0.03, 0.05$

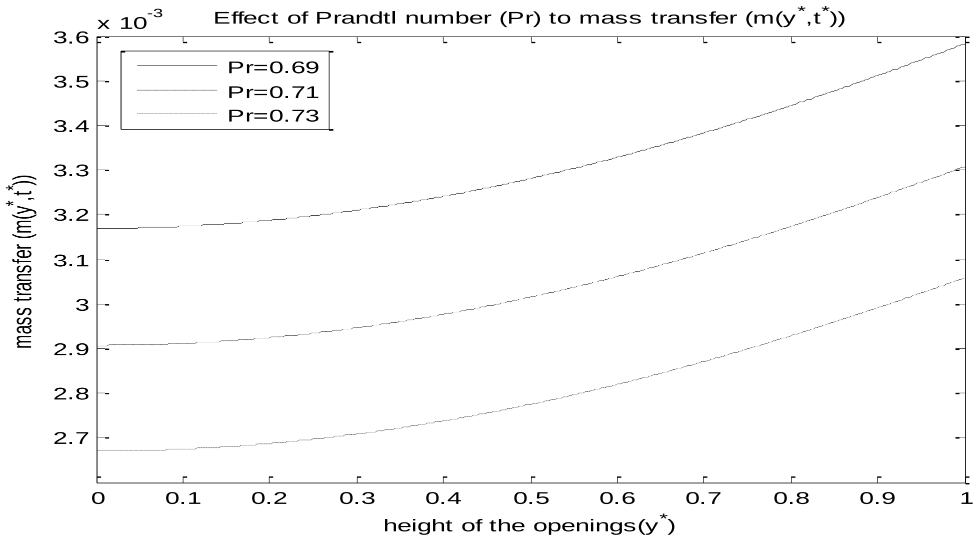

Fig.2.14: mass transfer $m$versus y^* and t^* for$Pr = 0.69, 0.71, 0.73$

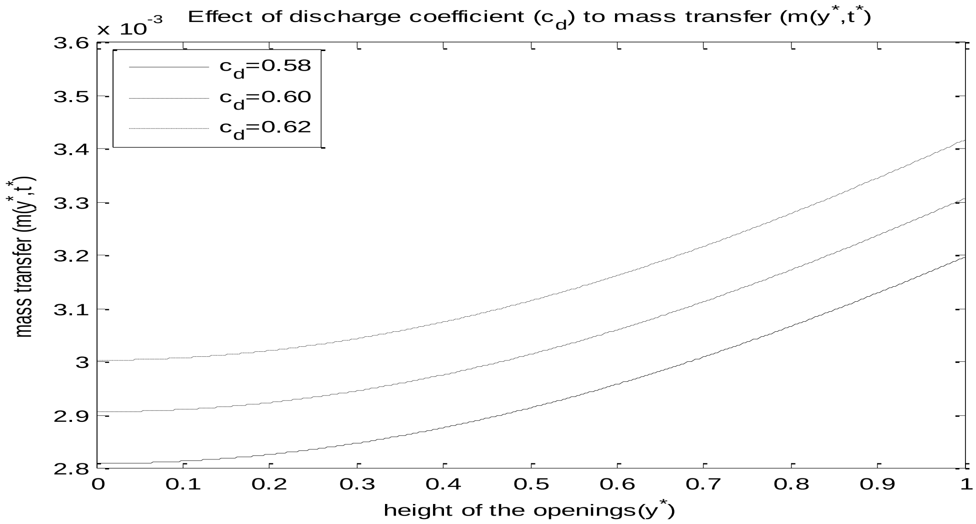

Fig.2.15: mass transfer m versus y^* and t^* for$c_d = 0.58, 0.60, 0.62$

III. DISCUSSION OF THE RESULTS

In this section the main features of the results obtained from the previous section will be discussed. The section intends to discuss the analyses of the results for velocity-, and temperature- profiles together with volumetric airflow and mass transfer in the domain.

Considering Fig. 2.3, it depicts the variation of fractional order α with the temperature of the air when the flow is induced by Stack driven forces. At Fig. 2.3,it can be observed that increasing the fractional order α between the intervals of $0 < \alpha < 1$ decreases the temperature profile across the openings. Therefore, the best value for optimal ventilation in the domain is when$\alpha = 0.1$. The increase of effective thermal coefficient (θ_0) at Fig. 2.4, shows a decrease in the temperature profiles. Therefore, the best value for optimal ventilation in the domain is when $\theta_0 = 0.01$.Fig. 2.4,it reveals that increasing the fractional order α decreases the velocity profiles across the openings. Therefore, the best value for optimal ventilation in the domain is when$\alpha = 0.1$. The increase of effective thermal coefficient (θ_0) at Fig. 2.7, shows a decrease in the velocity profiles within the domain envelope. Therefore, the best value for optimal ventilation in the domain is when $\theta_0 = 0.01$. Fig. 2.8, it reveals that increasing the Prandtl number (Pr)decreases the velocity profiles across the openings. Therefore, the best value for optimal ventilation in the domain is when$\mathrm{Pr} = 0.69$. Fig. 2.9,it reveals that increasing the fractional order α decreases the volumetric airflow in the domain. Therefore, the best value for optimal ventilation in the domain is when$\alpha =$

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0.1. Fig. 2.9, it reveals that increasing the effective thermal coefficient (θ_0)increases the volumetric air flow in the domain envelope. Therefore, the best value for optimal ventilation in the domain is when $\theta_0 = 0.05$. The increase of Prandtl number(Pr)at Fig. 2.11 shows and decrease in the volumetric air flow. Therefore, the best value for optimal ventilation in the domain is when$\mathrm{Pr} = 0.69$. The increase of discharge coefficient(c_d)at Fig. 2.12 shows an increase in the volumetric air flow. Therefore, the best value for optimal ventilation in the domain is when $c_d = 0.62$. Fig. 2.13,it reveals that increasing the fractional order α decreases the mass transfer from the domain. Therefore, the best value for optimal ventilation in the domain is when$\alpha = 0.1$. Fig. 2.14, it reveals that increasing the effective thermal coefficient (θ_0)increases the mass transfer from the domain envelope. Therefore, the best value for optimal ventilation in the domain is when $\theta_0 = 0.05$. The increase of Prandtl number(Pr)at Fig. 2.14 shows and decrease in the mass transfer. Therefore, the best value for optimal ventilation in the domain is when$\mathrm{Pr} = 0.69$. The increase of discharge coefficient(c_d)at Fig. 2.15 shows an increase in the mass transfer. Therefore, the best value for optimal ventilation in the domain is when $c_d = 0.62$.

IV. SUMMARY

In the thesis considerable research efforts have been made on understanding the airflow process through openings on vertical wall. In the literature on ventilation phenomena, a wealth of information is available and satisfactory means have been evolved for the estimation of the temperature profiles, velocity profiles, volumetric airflow and mass transfer in building envelopes, similar analytical procedures are available in the literature for a variety of combinations of boundary conditions, but not including the ones considered here.

The study considered a uniform interior temperature, the assumption has enabled the simplification of the ca n which approximation of reduced gravity was also calculation of air flow process in the domain. One dimensional Navier- Stokes equations was utilize, i invoked in order to maintain the effect of buoyancy forces in the domain envelope.

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V. CONCLUSION

The thesis studied the fractional time derivatives (Caputo type) of airflow process across vertical vents in rectangular building. The governing equations describing the flow are written in dimensionless form and solved analytically by Laplace transform technique. In order to ascertain the best for optimal ventilation. The effect of each physical parameters involved in the study are discussed with aid of plotted graphs. Therefore, In conclusion, the following have been achieved.

1. Increases of fractional order α between the intervals of $0 < \alpha < 1$ decreases the temperature profile across the openings.
2. Increases of effective thermal coefficient (θ_0) decreases the temperature profiles.
3. Increases of fractional order α decreases the velocity profiles across the openings.
4. Increases of effective thermal coefficient (θ_0) decreases the velocity profiles within the domain envelope.
5. Increases of Prandtl number (Pr)decreases the velocity profiles across the openings.
6. Increasing of fractional order α decreases the volumetric airflow in the domain.
7. Increases of effective thermal coefficient (θ_0)increases the volumetric air flow in the domain envelope.
8. Increase of Prandtl number(Pr)decreases the volumetric air flow.
9. Increases of discharge coefficient(c_d)increases of the volumetric air flow.
10. Increasing of fractional order α decreases the mass transfer from the domain.
11. Increases of effective thermal coefficient (θ_0)increases the mass transfer.
12. Increase of Prandtl number(Pr)decreases the mass transfer.
13. Increases of discharge coefficient(c_d)increases of the mass transfer from the domain.

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